Spiderplots versus Tornado Diagrams for Sensitivity Analysis

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Sensitivity analysis, supported by computer hardware and software, can easily overwhelm an analyst or decision maker with data. However, this data can be organized in a readily understandable way using well-designed graphs. Two graphical techniques, spiderplots and tornado diagrams, are commonly used respectively by engineering economists and decision analysts. Their advantages are complementary. Management scientists should often use both to convey their results to decision makers succinctly and clearly. The simpler tornado diagram can summarize the total impact of many independent variables. An individual spiderplot displays more information about a smaller number of variables. This includes the limits for each independent variable, the impact of each on the dependent outcome, and the amount of change required to reach a break-even point.

Quantitative models rely on data that is rarely exact. Current values must be estimated, and forecasting is required for future events, prices, needs, and opportunities. Even when deterministic models are better than stochastic ones, the uncertainty must be evaluated through sensitivity analysis. Often this is best done graphically. This sensitivity analysis may be used (1) to make better decisions, (2) to decide which data estimates should be refined before making a decision, or (3) to focus

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managerial attention on the most critical elements during implementation.

Sensitivity analysis can be defined as examining the impact of reasonable changes in base-case assumptions. I will discuss methods that can be applied to models with a single, dependent outcome and a number of independent variables. The question is the sensitivity of the dependent outcome relative to changes in each independent variable. This approach can be applied to many management science models since the single dependent outcome can be virtually any measure of the quality of the outcome. Examples include present worth, number of lives saved, market share in five years, and additive weighted multi-criteria functions.

Two disciplines within management science rely on specific graphic forms to convey the results of relative sensitivity analysis. Decision analysts rely on the tornado diagram [Howard 1988, Clemen 1991, and McNamee and Celona 1990] or on software that automatically generates tornado diagrams, and engineering economists rely on the spiderplot [Thuesen and Fabrycky 1989; and Eschenbach 1989]. Analysts in both fields and in many other fields have used both to study sensitivity analysis. Eschenbach and McKeague [1989] suggest that typical teaching and practice of engineering economists with respect to sensitivity analysis could be improved substantially. Other useful references on the graphical display of analytical data include Canada and Sullivan [1977] and Tufte [1983].

In doing a sensitivity analysis, the analyst should consider: (1) the reasonable limits of change for each independent variable, (2) the unit impact of these changes on the present worth or other measure of quality, (3) the maximum impact of each independent variable on the outcome, and (4) the amount of change required for each independent variable whose curve crosses over a break-even line.

Example Problem and Relative Sensitivity Analysis

Table 1 shows base-case values and defines the terms for a simple economic model of a prospective new product. For simplicity, I will assume that all cash flows except the first cost are end-of-year. Then, a single, deterministic, dependent outcome,

Variable	Lower Limit*	Base-Case Value	Upper Limit*
FirstCost	90%	$120,000	150%
Salvage	0%	$20,000	150%
N (horizon or life)	50%	12 years	200%
i (discount rate)	60%	10%	200%
O&M (annual cost)	80%	$6,000	125%
Revenue (annual)	60%	$55,000	125%
NNoRev (# years without revenue)	0%	1	300%
FracComp (fraction of revenue lost to competition)	0%	.2	200%

* Lower and upper limits are expressed as a percentage of the base-case values.

Table 1: A hypothetical set of project parameters defines an example where present worth is an appropriate measure of project acceptability.

the present worth is calculated as follows:

$$PW = -\text{FirstCost} + \text{Salvage} \cdot (P/F, i, N) - O\&M \cdot (P/A, i, N) + \text{Revenue} \cdot (1 - \text{FracComp}) \cdot (P/A, i, N - NNoRev) \cdot (P/F, i, NNoRev). \quad (1)$$

$(P/A, i, N)$ and $(P/F, i, N)$ are respectively the uniform periodic and the single payment present worth factors, where i is the interest rate, N is the number of periods, P occurs at time 0, A occurs at the end of periods 1 through N, and F occurs at the end of period N.

Columns 2 and 4 within Table 1 show the first step of sensitivity analysis, which is to define the limits of reasonable change, both plus and minus (or both upper and lower) for each independent variable. These limits are likely to differ for each variable, and they may be asymmetric since the worst case is often more extreme than the best.

Tornado diagrams and spiderplots are based on the equation(s) of the model (Eq. (1) for the example) and on relative sensitivity analysis. The base-case outcome (*PW* for the example) is the deterministic result. This defines the vertical axis of the tornado diagram and the center of the spiderplot. One at a time each variable is set to its upper and lower limits and the base-case outcome is calculated, while the other variables remain at their base-case values. This completes the computations for the tornado diagram. For the spiderplot, intermediate values must also be computed.

This example also illustrates how independence is used in sensitivity analysis for spiderplots and tornado diagrams. The number of years of no revenue and the fraction of the revenues lost to competition are assumed to be independent. However, delays that increased the number of years of no revenue would also tend to increase the fraction lost to competition. Dependencies like these require simulation or the construction of scenarios.

The Information in a Tornado Diagram or a Spiderplot

To construct a tornado diagram, Figure 1, one makes the independent variable whose limits have the widest range for the dependent outcome the top bar. Then one arrays the other variables in descending order of effect on the dependent outcome. The limits on these variables could be expressed as percent change, years, or dollars. A tornado diagram quickly highlights those variables to which the outcome is most sensitive. Such a diagram can include many variables, and it can also be constructed as a horizontal bar chart [Eschenbach 1989, Eschenbach and McKeague 1989].

To construct a spiderplot, Figure 2, one plots a curve for each variable on a single x-y plot. To avoid clutter, one must limit a

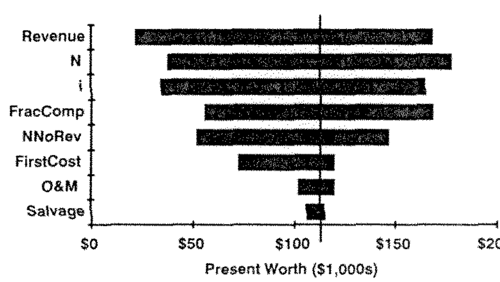

Figure 1: The tornado diagram for the example ranks the revenue variable as having the most impact on the present worth (salvage value has the least).

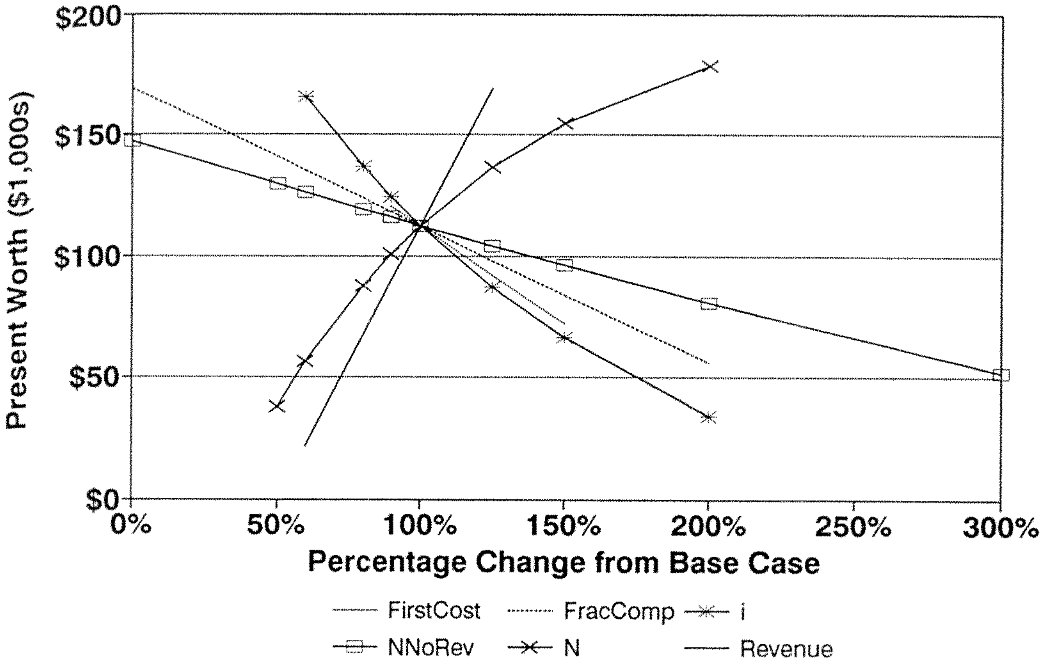

Figure 2: The spiderplot shows the relative change in present worth for reasonable changes in the independent variables. It also shows the upper and lower limits of expected changes in the independent variables.

single plot to about seven variables; four or five is better. For consistency, the x-axis measures each independent variable as a percentage of its base case. If an independent variable's base case is 0, then a second x-axis must be added to the plot, using the units of that variable, for example, dollars.

The spiderplot's greater complexity can convey more information (Table 2). A tornado diagram shows only (1) the outcome values (y-coordinates) at the ends of each spiderplot curve. The x-coordinates of these endpoints in the spiderplot curves depict (2) the limits for each independent variable. The slopes of the spiderplot curves depict (3) the relative change in the outcome for a unit change in the independent variable. The shape of the spiderplot curve also shows (4) whether linear or nonlinear relationships are present. (Nonlinearity can complicate interpreting tornado diagrams.) Tornado diagrams can be (5) easier to construct and can be constructed for more variables.

Common Errors

Because of their simplicity, tornado diagrams are quite easy to do correctly. However, a careless user might wrongly conclude that decreases in each independent variable are matched to decreases in the outcome. In the example shown in Table 1, decreases in first cost increase present worth.

Spiderplots are often drawn incorrectly [Eschenbach 1989]. Analysts should begin by defining reasonable limits for the independent variables, but they often use plus and minus the same arbitrary percentage. For example, zero to 200 percent might be

	Tornado Diagram	Spiderplot
Limits on each independent variable	Number not graph	Yes
Relative impact delta y/delta x	No	Yes
Total impact on outcome	Yes	Yes
Degree of change required for crossover	For outcome, not for independent variables	Yes
Total number of independent variables	Nearly unlimited	7 or less

Table 2: Tornado diagrams and spiderplots are designed to convey different information.

used for all variables. If this error is also common with tornado diagrams, it is less obvious.

This error is serious, because it extends each spiderplot curve to the left and right graph boundaries. On the tornado diagram, these "extended" y-coordinates would be plotted. The tornado diagram and the spiderplot would then (1) show the wrong extreme values for the outcome and (2) exaggerate the uncertainty for some independent variables. The tornado diagram would also (3) order the independent variables incorrectly.

A third error is to ignore uncertainty in the limits. Just as a positive present worth cannot determine a decision but must instead be weighed against noneconomic factors and the model's approximations; so too, must a break-even point be interpreted as a region of economic indifference. The apparent precision of numerical comparisons violates the imprecision, uncertainty, and intuitive feel of sensitivity analysis.

Linear versus Nonlinear Shapes and the Implications for Control

In spiderplots, most variables show (1) increasing or decreasing returns to scale (the slope's absolute value is increasing or decreasing), or (2) a straight line or proportional relationship to the outcome. In engineering economy, first costs, periodic payments or receipts, and other parameters found outside of the compound interest factors are usually related linearly to the present worth. Variables—such as the discount rate, inflation and other geometric gradients, a machine's life, or the problem's horizon—are inside the conversion factors, and they exhibit a decreasing returns-to-scale relationship to present worth.

In my experience, almost all independent variables show either a straight line or a concave or convex curved relationship to the dependent outcome. However, linearity or the lack of it is much more than an academic question. It affects the ability of the analyst or decision maker to respond to the uncertainty shown in the tornado diagram or spiderplot.

The total impact of the uncertainty linked to an independent variable has two sources—the upper and lower limits of that variable and the integrated slope of those changes on the outcome. Arguably the functional form of that relationship is likely to be uncontrollable. However, the limits are likely to be far more "controllable." In some cases, the analyst can obtain additional information. In other cases, managerial attention might better control key variables during implementation. Since the tornado diagram shows only changes in the dependent outcome and not in the independent variables; it suggests an incorrect implicit assumption—that each variable has the same proportional payoff for better control and the same opportunities for control.

Nonlinearity causes extra problems for tornado diagrams. In the example shown in Figure 2, a 10-percent change at the high end of project life or discount rate values has relatively little impact. This is apparent in the spiderplot, but the tornado diagram (Figure 1) seems to implicitly assume that a 10 percent reduction in either limit of an independent variable will change the outcome proportionately.

Sensitivity analysis is examining the impact of reasonable changes in base-case assumptions.

In the example shown in Figure 1, only the curves for revenue, project life, and discount rate approach the break-even value of present worth which equals $0. If a curve crosses the break-even line, then it is helpful to describe the percentage increase or decrease in that variable needed to reach the break-even point.

Conclusion

Both tornado diagrams and spiderplots are useful in assessing the impact of uncertainty. This sensitivity analysis can improve decision making, point out needed refinements in data estimates, and focus managerial attention. But to use them most effectively, the analyst must use both graphs and must consider how best to "draft" them. Depending on the software available and the individual's experience, the analyst might find either easier to do—but the key question is which is right for the information and the audience.

The tornado diagram highlights those variables meriting further attention and summarizes the total impact of each variable. However, only the spiderplot can show which of the high-impact variables are likely to be amenable to control through specific managerial action or through further data gathering. Only the spiderplot shows all of the following: (1) the reasonable limits of change for each independent variable, (2) the unit impact of these changes on the dependent outcome, (3) the maximum impact of each variable on the dependent outcome, and (4) the amount of change required to cross-over the break-even line. The tornado diagram shows only (3) the total impact, but it can do so for many more variables.

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